EULER's METHOD: SAMPLE CALCULATION

1. Equation to be solved must be in the following form (Eq-1):

$$\frac{dy}{dt} = f(t,y); \quad y(t_o) = y_o; \quad t_o \leq t \leq t_n$$

2. Apply the Euler's approximation (Eq-2):

$$y_i = y_{i-1} + hf(t_{i-1}, y_{i-1})$$

3. Time points can be computed by several equations (recall that $\Delta t = h$) (Eq-3a, Eq-3b):

$$t_i = t_{i-1} + h$$ where i=2,3,… t_n

$$t_i = t_1 + (i-1)h$$ for i=2,3,…t_n where t_1 is the initial time (initial condition)

4. An initial condition must be specified (Eq-4):

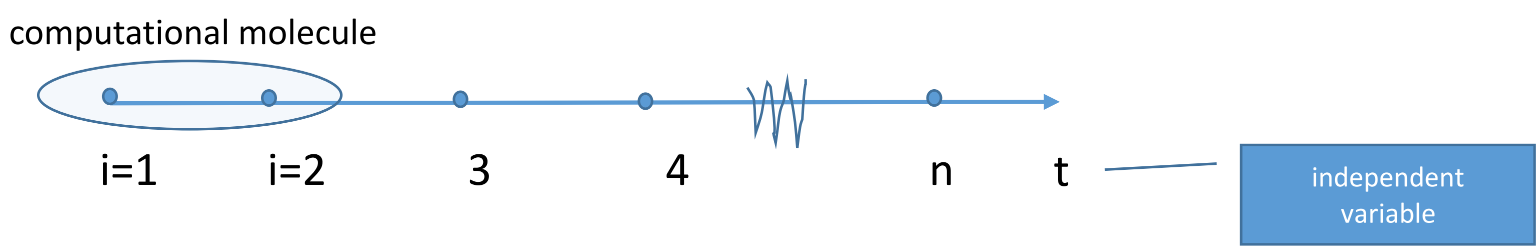
t_o *becomes* t_1 and $y(t_o)$ *becomes* $y(t_1) = y_1;$

The left hand side is the usual Math notation and the right-hand side has been prepared for MATLAB coding in which array indices start at 1

5. Specified the range of the independent variable where the solution is desired (discretization) and the number of points **n**. The number of points will depend on the value of the step size. Larger the step size less points to be solved and vice versa.

Math: $t_o \leq t \leq t_n;$ Matlab: $t_1 \leq t \leq t_n$

For instance if we would like to solve for h=0.2 in the range $(t_1 = 0) \leq t \leq (t_n = 2)$ then n = [(t_n-t_1)/h]+1= [(2-0)/0.2]+1 = 10+1=11 points.



6. You start the solution with Eq-3 and computes all values of t_i (independent variable). Next you compute each value of y_i with Eq-2

7. You construct the solution as a table of t vs y and graph it. The solution is presented as discrete points (no a continuous line). We usually take advantage of graphing packages in which we can draw a solid or dashed line behind the points, showing the "trend" of the solution.

8. Now you solve the equation again with Euler's for several smaller h's. When there are no appreciable changes in the solution, i.e., points overlap, then you are done.

9. EXAMPLE: Apply Euler's method to solve

$$\frac{dy}{dt} + 2y - t = 0\,;\;\; y(0) = 1;\;\; 0 \leq t \leq 2$$

Rearrange the problems as:

$$\frac{dy}{dt} = \underbrace{t - 2y}_{f(t,y)} \; ; \;\; y(0) = 1; \;\; 0 \le t \le 2$$

10. Chose h=0.2, then n = [(t_n-t_1)/h]+1= [(2-0)/0.2]+1 = 10+1=11 points in the range 0<=t<=2.

11. Expand Eq-2 and Eq-3 for i=2,3,…n, and construct a table:

$$y_i = y_{i-1} + h(t_{i-1} - 2y_{i-1})$$

i (array index and computation counter)	**t_i**	**y_i**
1➔ initial condition➔	T1=0	Y1=1
2	T2=T1+H=0+0.2=0.2	Y2=Y1+H(T1-2Y1)=1+0.2(0-2*1)=0.6
3	T3=T2+H=0.2+0.2=0.4	Y3=Y2+H(T2-2Y2)=0.6+0.2(0.2-2*0.6)=0.4
4	T4=T3+H=0.4+0.2=0.6	Y4=Y3+H(T3-2Y3)=0.4+0.2(0.4-2*0.4)=0.32
5	T5=T4+H=0.5+0.2=0.8	…
…	…	…
…	…	…
11	T11=T10+H=1.8+0.2=2.0	Y11=Y10+H(T10-2Y10)

EXCEL

Using Excel to find the solution for h=0.2

To calculate time you have several equations:
(1) t(ii)=t(ii-1)+h
(2) t(ii)=t(1)+(ii-1)*h

h=	0.2	
ii	**t(ii)**	**y(ii)**
1	0	1
2	0.2	0.6
3	0.4	0.4
4	0.6	0.32
5	0.8	0.312
6	1	0.3472
7	1.2	0.40832
8	1.4	0.484992
9	1.6	0.570995
10	1.8	0.662597
11	2	0.757558

Formulas:

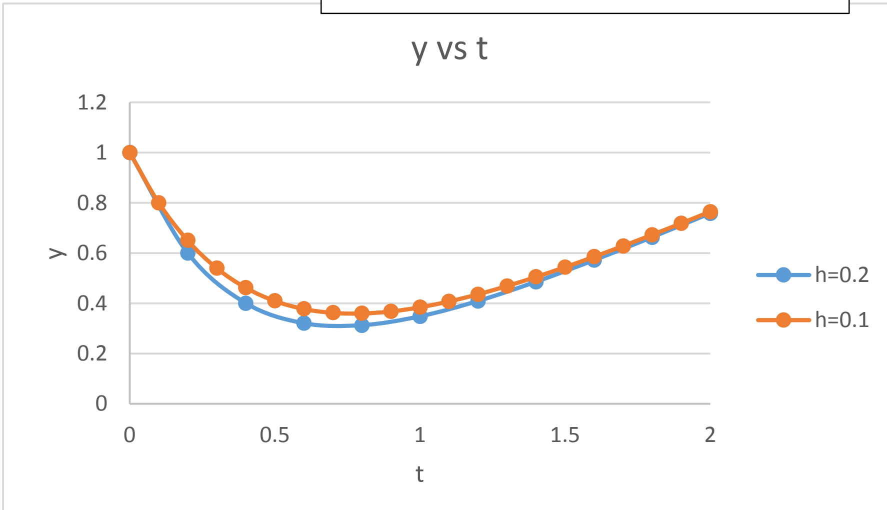
`'=$D$15+(C16-1)*0.2`

`'=E15+$D$12*(D15-2*E15)`

Do it again for smaller h

h=	0.1	
ii	**t(ii)**	**y(ii)**
1	0	1
2	0.1	0.8
3	0.2	0.65
4	0.3	0.54
5	0.4	0.462
6	0.5	0.4096
7	0.6	0.37768
8	0.7	0.362144
9	0.8	0.359715
10	0.9	0.367772
11	1	0.384218
12	1.1	0.407374
13	1.2	0.435899
14	1.3	0.468719
15	1.4	0.504976
16	1.5	0.54398
17	1.6	0.585184
18	1.7	0.628147
19	1.8	0.672518
20	1.9	0.718014
21	2	0.764412

Graph both solutions. Keep solving and graphing until the solutions overlap



Solutions for several decreasing values of h=[0.2,0.1,0.05,0.025...]. You can see that the solution with h=0.025 starts ovelapping all its points.