# INGE4035. Numerical Methods Applied to Engineering

FIRST EXAMINATION- Nov 30, 2017. Show enough details and assumptions for each problem answer. Place your solution exactly in the space provided in the answer sheet. Handle numbers with up to 4 decimal places and methods with running index starting in zero.

## Problem ONE: 1st order IVP-ODE:

| | |
|---|---|
| Consider the following problem: $$\frac{dY}{dt} - t^{2Y} = 0$$ $$Y(t=0) = 1, \quad t = [0,1]$$ | Starting with $h = \Delta t = 0.20$ and $t_0 = 0$ compute the solution for TWO times steps, $t_1$, and $t_2$. The solution at each time step involves to find the pair of values corresponding to $t_i$ and $Y_i$. |

Use the Midpoint 2nd order method, which is defined as follows:

| | |
|---|---|
| $$K_1 = f(t_i, Y_i)$$ $$K_2 = f(t_i + \frac{h}{2}, Y_i + \frac{h}{2}K_1)$$ $$Y_{i+1} = Y_i + hK_2$$ | (A) Discretize the domain of the problem.<br>(B) Express the ODE and its IC in standard format suitable for a solution with the Midpoint Method: |

(C) Show explicit computation for the first time step (i.e., $t_1$). Explicit computation implies to write the formula (pay attention to indices), plug in the numbers, and show the result, in that order.

(D) Show explicit computation (formulas + plug-in numbers + results; in the right sequence) for the second time step (i.e., $t_2$) :

(E) Arrange the solution so far as a Table with columns: **i** (running index & counter), $t_i$, $K_1$, $K_2$, $Y_i$.

## Problem TWO. Higher-Order ODE

Consider the system of coupled IVP-ODEs given by:

$$x'' = t + x' + y', \qquad x(0) = 1, x'(0) = 2$$

$$y''' = x'y'' + x, \qquad y(0) = -1, \; y'(0) = 1, \quad y''(0) = 2$$

$$0 \le t \le 1$$

Convert the system of higher-order equation to a system of first-order equations. To help understand your work, please use for, starting with the first ODE, the new variables convention A, B, C, and so on, to reduce the order of both equations.

(A) How many first order equations are generated?

(B) Show each step in the conversion of the main equations:

(C) Show next the conversion of the ICs to new auxiliary variables:

| Original | Transformation |
|---|---|
| $x(0) = 1$ | |
| $x'(0) = 2$ | |
| $y(0) = -1$ | |
| $y'(0) = 1$ | |
| $y''(0) = 2$ | |

(D) Express clearly each equation with its corresponding initial condition (IC) in a Table:

| **ODE's** | **IC's** |
|---|---|
| | |
| | |
| | |
| | |
| | |

## Problem THREE. BVP-ODE

For the problem:

$$\frac{d^2u}{dt^2} = 1, \qquad u'(0) = 0.4 \quad u(1) = 2$$

Chossing $\Delta t = 0.25$ show how to numerically solve the BVP-ODE using the Finite Difference-Tridiagonal Matrix Method. Follow the following procedure

(A) Discretize the domain of the problem
(B) Discretize the Ordinary Differential Equation and find the recurrence formula
(C) Discretize the Boundary Conditions. If you need to approximate the first derivative in the Boundary Condition, use a derivative approximation of error order $h^2$.
(D) Develop the system of equations needed to solve the problem
(E) Express the system of equations in Matrix form.