

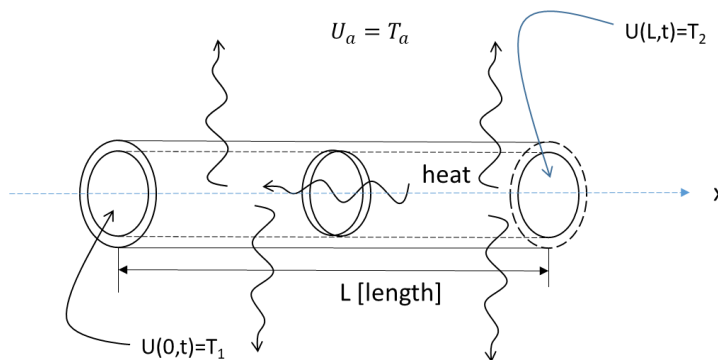
Last name _____ First name _____ ID _____

PROBLEM#1. FTCS PDE. Consider the Heat Equation below,

$$\frac{\partial U}{\partial t} = k \frac{\partial^2 U}{\partial x^2} + h'(U_a - U) \quad [Eq 1]$$

as the equation governing the temperature distribution along a thin rod with heat exchange with its surrounding and fixed temperatures at its ends with the following specifications:

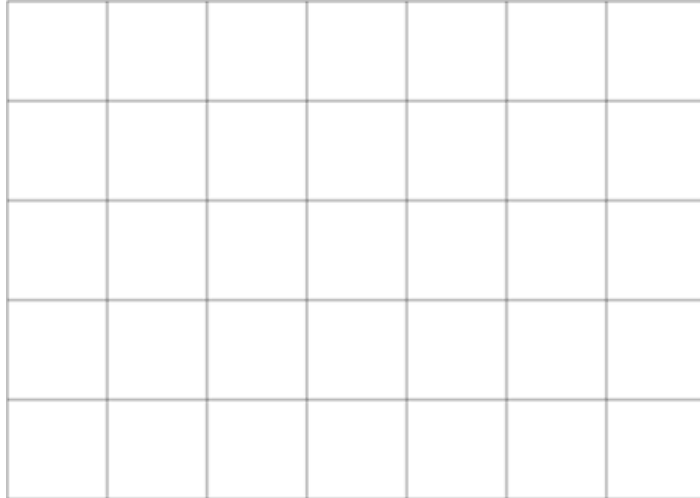
- $L = 5 \text{ cm}$ (rod length)
- $\Delta x = 1 \text{ cm}, \Delta t = 0.1 \text{ s}, k = 0.835 \text{ cm}^2/\text{s}$ (thermal diffusivity), and
- $\lambda = k\Delta t / \Delta x^2 = 0.835(0.1) / (1)^2 = 0.0835$ (this parameter may shows later along the discretization of the ODE)
- $U_a = T_a = 20$ [air temperature]
- $T_1 = 40 \text{ }^\circ\text{C}$
- $T_2 = 200 \text{ }^\circ\text{C}$
- $h' = 0.01 \text{ m}^{-2}$ [heat transfer coefficient, parameterizes the rate of heat dissipation to the surrounding air]



At $t = 0$, the temperature of interior nodes of the rod is zero and the boundary conditions are

- IC: $U(x,0) = 0 \text{ }^\circ\text{C}$ for $0 < x < L$
- BC: $U(0,t) = T_1 \text{ }^\circ\text{C}$ for $t > 0$
- BC: $U(L,t) = T_2 \text{ }^\circ\text{C}$ for $t > 0$

(1) **Discretize the domain of the problem.** For two independent variables use a mesh grid, each axis representing one of the axes. Use as running indices, i for space and k for time, e.g., $i=0,1,2,\dots,ix$, and $k=0,1,2,\dots,kx$. Both starting in zero.



(2) Discretization of the Differential equation.

Starting with

$$\frac{\partial U}{\partial t} = k \frac{\partial^2 U}{\partial x^2} + h'(U_a - U) \quad [Eq 1]$$

To develop the FTCS scheme:

- a. Approximate the 1st order time derivative with _____ finite difference approximation of $\partial(\Delta t^n)$ [$n=$ _____] at the grid point (i,k) :

- b. Evaluate $\left. \frac{\partial^2 U}{\partial x^2} \right|_{i,k}$ with _____ finite difference approximation of $\partial(\Delta x^n)$ [$n=$ _____]

- c. Evaluate U at the grid point (____,____)

- d. Find the recurrence formulation grouping characteristic parameters, e.g., $\lambda = \frac{\kappa \Delta t}{\Delta x^2}$

(3) Discretize the initial and boundary conditions.

Original Mathematical Expression	Discretized
$U(0, t) = T_1$ for $t > 0$	
$U(L, t) = T_2$ for $t > 0$	
$U(x, 0) = 0$ for $0 < x < L$	

(4) Show the FTCS computational molecule (CM) within the grid of your discretization in question-1.

see GRID above

Answer the following questions:

- a) How many unknowns do you have at each time step? _____
- b) How many times does the computational molecule fit in the grid during the computation of one time step? _____

(5) System of Equations. According to your problem discretization and the number of unknowns find the appropriate number of equations to solve for the FIRST time step (i.e. $[t_1, U_i^1]$). We use indexed-variable algebraic equations—Not numbers yet. These equations should be presented before and after including the IC & BC.

- a. Before including IC & BC

- b. After including IC & BC

(6) Express the problem with numbers for a clean system of linear equations and also in matrix form.

a. System of equations:

b. Matrix Form

(7) Solve either one (6.a) Excel or (6.b) matlab for $0 \leq t \leq 1.0$